Problem Set 7

① The implicit strike price is $0.75 per CAD.

$$d_1 = \frac{\ln(0.72/0.75) + (0.045 - 0.0275 + \frac{1}{2}0.05^2) \times 6/12}{0.05\sqrt{6/12}}$$

$$= -0.889 \approx -0.89$$

$$\phi(d_1) = 0.1922$$

$$d_2 = -0.889 - 0.05\sqrt{6/12} = -0.924$$

$$\phi(d_2) = 0.1788$$

$$C = 10{,}000{,}000\,(0.72e^{-0.0275\times 6/12} \times 0.1922 - 0.75e^{-0.045\times 6/12} \times 0.1788)$$

$$= \$53{,}778$$

$$P = 53{,}778 + 10{,}000{,}000\,(-0.72e^{-0.0275\times 6/12} + 0.75e^{-0.045\times 6/12})$$

$$= \$285{,}235.$$

(2) $$d_1 = \frac{\ln(70/72.5) + \frac{1}{2}\, 0.30^2 \times 6/12}{0.30\sqrt{6/12}} = -0.059$$

$$d_2 = -0.059 - 0.3\sqrt{6/12} = -0.271$$

$$\phi(d_1) = 0.4721 \qquad \phi(d_2) = 0.3936$$

$$C = 70\, e^{-0.05 \times 6/12} \times 0.4721 - 725\, e^{-0.05 \times 6/12} \times 0.3936$$

$$= 4.40.$$

(3) We can price the option as if it is a futures option.

$$d_1 = \frac{\ln(86225/86000) + \frac{1}{2}\, 0.80^2 \times 3/12}{0.80\sqrt{3/12}} = 0.207$$

$$d_2 = 0.207 - 0.8\sqrt{3/12} = -0.193$$

$$\phi(d_1) = 0.5910 \qquad \phi(d_2) = 0.4325$$

$$C = 86225\, e^{-0.045 \times 3/12} \times 0.5910 - 86000\, e^{-0.045 \times 3/12} \times 0.4325$$

$$= \$13,610$$

(4) A futures requires no investment. In a risk-neutral world, if the drift was positive, every trader would buy the futures, pushing the price up and therefore the drift down. The opposite would happen if the drift was negative. Thus, the only drift consistent with equilibrium is zero.

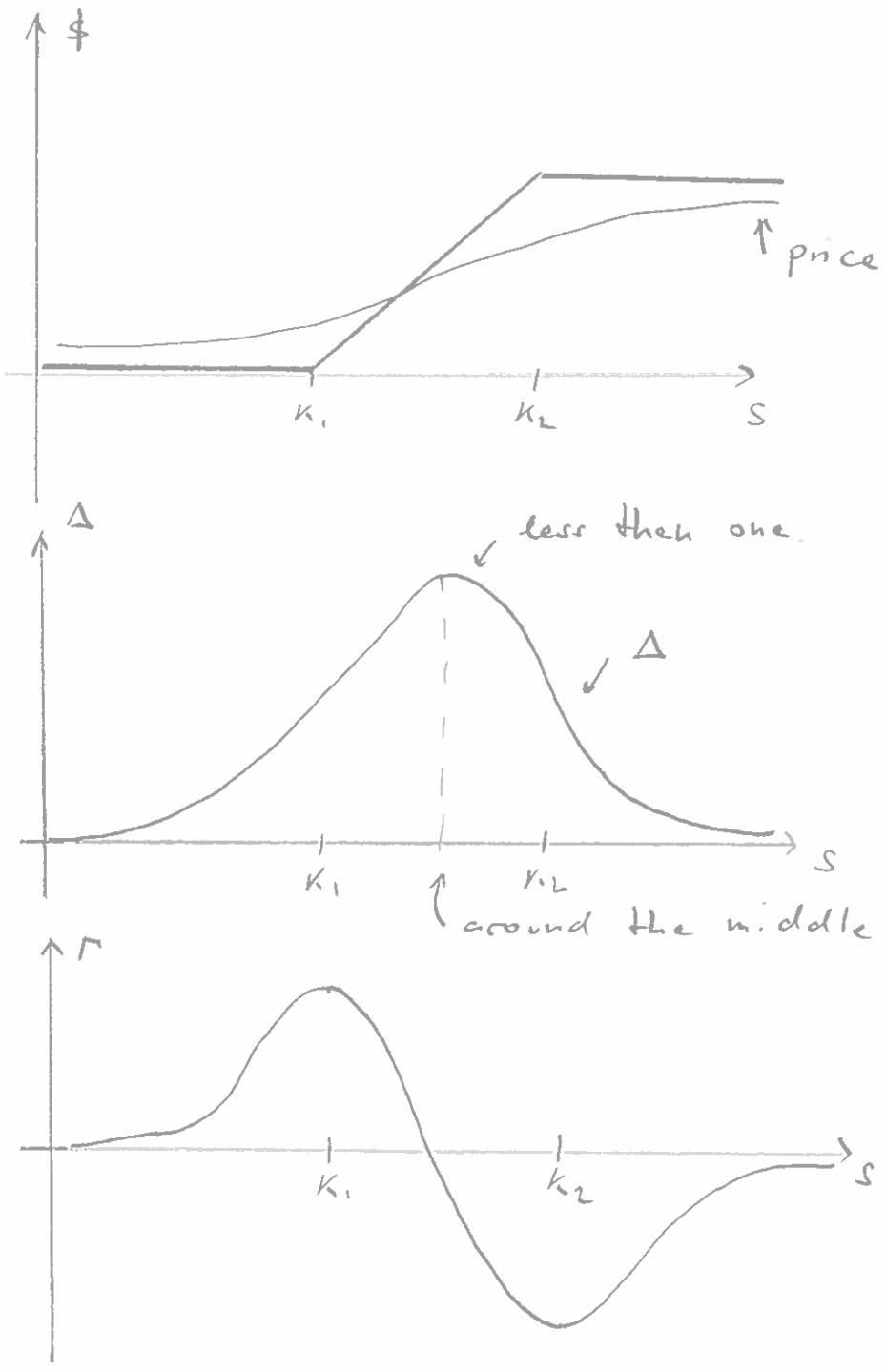
(5) Assuming 252 trading days per year, a theta of -0.2 means a decay of

$$\frac{-0.2}{252} = -0.08 \text{ b.p. per trading day.}$$

If the trader wants to profit from the stock moving in any direction, a delta neutral position with positive gamma such as ATM straddle would be appropriate. The risk for the trader is that if the stock does not move she will lose money due to the gamma being negative.

⑥ If gamma is large and positive and the delta is zero, the theta will be large and negative. Unless the stock moves significantly in any direction, the trader will "bleed" theta.

⑦



③ $\Delta_{portfolio} = -2{,}000 \times 0.5 + 800 \times 0.8 - 2{,}000 \times (-0.4) + 1{,}500 \times (-0.6$

$= -460$

$\Gamma_{portfolio} = -2{,}000 \times 0.008 + 800 \times 0.020 - 2000 \times 0.012 + 1500 \times 0.007$

$= -13.5$

$V_{portfolio} = -2000 \times 0.40 + 800 \times 0.05 - 2{,}000 \times 0.12 + 1{,}500 \times 0.42$

$= -320.$

a. You should buy the traded option to offset the negative gamma of your portfolio.

$$N = \frac{13.5}{0.011} = 1{,}227.27 \text{ options}$$

The new delta is $-460 + (-0.4) \times 1{,}227.27 = -951$ so buy €951.

b. You should buy the traded option to offset the negative vega of your portfolio.

$$N = \frac{320}{0.08} = 4{,}625 \text{ options}$$

The new delta is $-460 + (-0.4) \times 4{,}625 = -2{,}310$ so buy €2,310.

(9)

a. A short straddle is concave at the strike price, thus its theta is positive.

b. A long strangle is convex between the strikes, so its theta is negative.

c. A bull spread is concave at K_2, so its theta is positive

d. A bear spread is convex at K_2, so its theta is negative.